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GEOMAGNETIC CONTROL OF DIFFUSION IN THE UPPER ATMOSPHERE

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ABSTRACT

In many recent papers concerned with providing an explanation for the geomagnetic anomaly, favorable agreement with measured data has been obtained from the equations of motion for electrons and ions when used with an empirical boundary condition, whereas poor agreement has resulted from attempts to numerically integrate the diffusion equation derived from the equations of motion. We have been able to demonstrate that the derivation ~~deviation~~ of the commonly employed diffusion equation is based on assumptions concerning the equations of motion which are inconsistent with the observed distribution unless an additional constraint equation is also applied. Unfortunately, application of the constraint equation also provides incorrect results. We find, therefore, that none of the currently used forms of the diffusion equation which take ~~taking~~ geomagnetic control into account provide correct results because they are based on improper physical models for the ionosphere.

Since the equations of motion do provide a favorable description for the geomagnetic anomaly, we have studied the possible physical models leading to the form of the equations used, and found that although field aligned diffusive equilibrium provides the correct form, a more reasonable assumption

concerning electron and ion collisions with neutrals also leads to the same result. We have also been able to provide a more realistic theoretical description of the geomagnetic anomaly by employing an analytic form for the boundary condition which is more accurate with measurement than those previously used.

Finally, by combining the equations of motion for neutrals, electrons and ions, we have been able to predict geomagnetic control for the neutral atmosphere in the lower F region of the ionosphere, although the exact shape of this distribution is unknown.

GEOMAGNETIC CONTROL OF DIFFUSION IN THE UPPER ATMOSPHERE

Introduction

In recent months it has become increasingly evident that a certain amount of confusion exists in the understanding of the basic physical mechanisms governing diffusion and the existence of the geomagnetic anomaly in the ionosphere. This apparent confusion arises by comparison of the work of Chandra (1964), (to be referred to as C-I) in which it is shown that the assumption of ambipolar diffusion along a field line cannot lead to geomagnetic control of the charged particles, and such papers as Goldberg and Schmerling (1962, 1963), (to be referred to as GS-I and GS-II) and Goldberg, Kendall, and Schmerling (1964), (to be referred to as GKS) in which this process does appear to produce geomagnetic control of the charged particle density in the ionosphere. It is our contention that these two apparently opposite view points do not contradict each other and that the confusion is almost entirely one of terminology usage and comprehension of the fundamental physics governing the derived equations.

The purpose of this paper is to describe and resolve the confusion which exists in the field at this moment, and then to point out the new problem with which we must contend in order to derive and apply the diffusion equation to ionospheric problems correctly. In addition, a section will be devoted to an improved theoretical description of the geomagnetic anomaly, by using an analytic expression for the vertical electron density distribution at the equator which is more in accordance with measurement than the simple Chapman type distrubution employed in GKS.

Fundamental Equations and Definitions

The major cause of confusion appears to lie in the application of two phrases, viz. ambipolar diffusion and diffusive equilibrium. Let us investigate and discuss each of these terms to determine how loose usage of them has led to the current problems of misunderstanding.

In the normal sense, ambipolar diffusion refers to a plasma in which the negative (electrons) and positive (ions) charges do not move independently due to the influence of the electric field caused by their Coulomb interactions. In this medium, the electrons and ions drift in pairs and this motion of electron-ion pairs is referred to as ambipolar diffusion. The condition for ambipolar diffusion in a neutral plasma is thereby

$$\vec{v}_e = \vec{v}_i = \vec{v} \qquad (1)$$

where $\vec{v}$ is macroscopic velocity and the subscripts e and i refer to electrons and ions respectively. When

$$\vec{v} = 0 \qquad (2)$$

the condition for diffusive equilibrium is ~~then~~ satisfied.

The implications of (1) are quite straight forward, as shown in C-I. In an isothermal atmosphere and in the presence of magnetic field, this requires generally $\nabla \times (\vec{v} \times \vec{B}) = 0$. In particular, the assumption of field aligned plasma diffusion ($\vec{v} \times \vec{B} = 0$) can only be satisfied for a trivial case, $\vec{v} = 0$, resulting in hydrostatic distribution of electron density independent of geomagnetic latitude.

On the other hand, favorable comparison between Alouette topside sounder measurements and theoretical calculations of the geomagnetic anomaly has been obtained in GKS by assuming conditions of ambipolar diffusion and diffusive equilibrium along field lines, thereby indicating a possible conflict with the results in C-I. The problem resolves itself once one investigates the meaning of ambipolar diffusion and diffusive equilibrium in the GKS sense.

Let us first write the general equations of motion for neutrals, electrons and ions, respectively, where the subscript n refers to neutrals. Following C-I:

$$\frac{n_e m_e m_n}{m_e + m_n} \nu_{en} (\vec{v}_n - \vec{v}_e) + \frac{n_i m_i m_n}{m_i + m_n} \nu_{in} (\vec{v}_n - \vec{v}_i) = -\nabla p_n + n_n m_n \vec{g} \qquad (3)$$

$$\frac{n_i m_e m_i}{m_e + m_i} \nu ei (\vec{v}_e - \vec{v}_i) + \frac{n_e m_e m_n}{m_e + m_n} \nu en (\vec{v}_e - \vec{v}_n) = -\nabla p_e + n_e m_e \vec{g} \qquad (4)$$

$$-e\, m_e (\vec{E} + \vec{v}_e \times \vec{B})$$

$$\frac{n_i m_i m_e}{m_e + m_i} \nu ei (\vec{v}_i - \vec{v}_e) + \frac{n_i m_i m_n}{m_i + m_n} \nu in (\vec{v}_i - \vec{v}_n) =$$

$$-\nabla p_i + n_i m_i \vec{g} + e n_i (\vec{E} + \vec{v}_i \times \vec{B}) \qquad (5)$$

where n is number density, $\nu_{k\ell}$ is the collision frequency between the k^{th} and ℓ^{th} particle, m is mass, p is pressure, $\vec{g}$ is gravitational acceleration, e is the absolute value of electron charge, $\vec{E}$ is electric field, and $\vec{B}$ is magnetic field, In writing equations (3) - (5) it is assumed that $\frac{\nu_{ek}}{n_k} = \frac{\nu_{k\ell}}{n_\ell}$.

In the following we assume that the plasma is in a quasi-neutral state

$$n_e \approx n_i = N \tag{6}$$

and the electrons, ions and neutrals obey the ideal gas law in the ionosphere,

$$p_j = n_j k T_j \tag{7}$$

when k is Boltzmann's constant and T is temperature. Furthermore, we assume thermal equilibrium, i.e.

$$T_e = T_i = T \tag{8}$$

Then,

$$p_e = p_i = p \tag{9}$$

In addition, we assume for simplicity that

$$\vec{v}_n \approx 0 \tag{10}$$

Then summation of (4) and (5) provides

$$\frac{m_n m_e N}{m_e + m_n} \nu_{en} \vec{v}_e + \frac{m_n m_i N}{m_i + m_n} \nu_{in} \vec{v}_i = -2\nabla p + N (m_e + m_i) \vec{g} + \vec{J} \times \vec{B} \tag{11}$$

where $\vec{J}$ is current density, defined as

$$\vec{J} = Ne (\vec{v}_i - \vec{v}_e) \tag{12}$$

Since we are investigating ambipolar diffusion and diffusive equilibrium in the GKS sense, it is desirable to write this equation in component form along a field line as

$$\frac{m_n m_e}{m_e + m_n} \nu_{en} \vec{v}_e \cdot \hat{B} + \frac{m_n m_i}{m_i + m_n} \nu_{in} \vec{v}_i \cdot \hat{B} = \left[\frac{-2kT\nabla N}{N} + (m_e + m_i)\vec{g}\right] \cdot \hat{B} \tag{13}$$

when $\hat{B}$ is a unit vector in the direction of the magnetic field. Let us write (13) in more familiar form by using

$$m_e << m_i, m_n \tag{14}$$

and defining the scale height of the ionizable constituent as H_i where

$$H_i = \frac{kT}{m_i g} \tag{15}$$

Also, for convenience, we make the approximation

$$m_n \approx m_i \tag{16}$$

Then

$$\frac{m_e \nu_{en}}{2} \vec{v}_e \cdot \hat{B} + \frac{m_i}{4} \nu_{in} \vec{v}_i \cdot \hat{B} = -kT \left(-\frac{\nabla N}{N} + \frac{\hat{g}}{2H_i}\right) \cdot \hat{B} \tag{17}$$

Finally, we write

$$\hat{B} = -(\hat{i}_r \sin I + \hat{i}_\theta \cos I) \tag{18}$$

where $\hat{i}_r$ and $\hat{i}_\theta$ are unit vectors in the r and θ directions and I is the magnetic dip angle, reckoned positive when the north seeking pole of the needle points downward. Now, if we treat ambipolar diffusion in the GKS sense, we simply imply that the electron and ion velocity components in the field direction are equal, i.e.

$$\vec{v}_e \cdot \hat{B} = \vec{v}_i \cdot \hat{B} = v_{11} \tag{19}$$

Applying (18) and (19) in (17), we obtain

$$v_{11} = \frac{-kT}{\mu\nu}\left[\sin I \left(\frac{1}{N}\frac{\partial N}{\partial r} + \frac{1}{2H_i}\right) + \frac{\cos I}{Nr}\frac{\partial N}{\partial \theta}\right] \tag{20}$$

where

$$\mu\nu = \frac{m_e \nu_{en}}{2} + \frac{m_i \nu_{in}}{4} \tag{21}$$

Assuming that

$$m_e \nu_{en} << m_i \nu_{in} \tag{22}$$

we may write

$$\mu\nu \approx \frac{m_i \nu_{in}}{4} \tag{23}$$

because of (14).

Equation (20) is a familiar result derived in such papers as Kendall (1962) and GS-II. However, it is clearly not the result of ambipolar diffusion, which is given by (1), but instead, the result of a statement concerning the field line components of electron and ion velocities given by (19).

If we now demand

$$v_{11} = 0 \tag{24}$$

which is the statement implying diffusive equilibrium along a field line in the GKS sense, we obtain the familiar equation

$$\sin I \left(\frac{1}{N}\frac{\partial N}{\partial r} + \frac{1}{2H_i}\right) + \frac{\cos I}{Nr}\frac{\partial N}{\partial \theta} = 0 \tag{25}$$

which can also be written as

$$\frac{1}{N}\frac{dN}{dr} + \frac{1}{2H_i} = 0 \tag{26}$$

provided we recognize that r and θ are not independent in (25) but related by the dipole field condition

$$r = r_o \sin^2 \theta \tag{27}$$

$$\text{and } \tan I = 2\,\mathrm{co}\overset{t}{\cancel{\mathrm{s}}}\,\theta \tag{28}$$

It is evident that (26) can only be treated in this total derivative form if the integration is carried out along the field line.

Statements concerning the components of vectors in a particular direction, such as (19), do not imply any conditions on the total vector. As a result, (25) has not required the assumption of any restrictions on the behavior of the velocity components normal to the field lines.

Equation (26) has been the basis of describing geomagnetic control in the upper F-region in GKS paper. Although the results of this paper appear justified on the grounds that diffusive equilibrium occurs along a field line, it is nevertheless undesirable to apply a concept which is of purely hypothetical nature. We now investigate other assumptions to find a more realistic justification for (25).

Consider equation (17). If we apply (19), we obtain

$$\frac{m_e \nu_{en}}{2} v_{e_{11}} + \frac{m_i}{4} \nu_{in} v_{i_{11}} = -kT \left(-\frac{\nabla N}{N} + \frac{\hat{g}}{2H_i}\right) \cdot \hat{B} \tag{29}$$

$m_i \nu_{in}$

We find two ways in which the right hand side will approach zero. The first imposes a new condition on the velocities, viz:

$$\vec{v}_e = \frac{-m_i \nu_{in}}{2m_e \nu_{en}} \vec{v}_i \qquad \vec{v}_e = -\frac{m_i \nu_{in} \vec{v}_i}{2m_e \nu_{en}}$$

or (30)

$$v_{e_{11}} = \frac{-m_i \nu_{in}}{2m_e \nu_{en}} v_{i_{11}} \qquad v_{e_{11}} = -\frac{m_i \nu_{in} v_{i_{11}}}{2m_e \nu_{en}}$$

a result which, although possible, appears rather unlikely since it would require a very special condition that the electron velocity be of the order of 10^3 times greater and in the opposite direction than the ion velocity.

However, if we can demand that the collision frequencies between electron and neutral and ion and neutral be sufficiently small so that the drag forces arising due to collision be negligible as compared to the pressure gradient, gravity and Lorentz forces, it is possible to derive equation (~~36~~ 26) without imposing any restriction on the velocities of electron and ion. We believe that this assumption is more realistic in the upper F-region where the gyro-frequencies of electron and ion are much greater than their corresponding collision frequencies.

Although the collision frequency assumption is physically more desirable, it prevents us from obtaining a simple expression for $v_{e_{11}}$ or $v_{i_{11}}$. Instead, we must return to the original equations of motion, (4) and (5), and solve for $\vec{v}_e$ and $\vec{v}_i$ explicitly, as has been carried out in the appendix in C-I. Unfortunately this introduces a very serious complication in the work because of the difficulty of eliminating electric field from the expressions of $\vec{v}_e$ and $\vec{v}_i$ without making specific assumptions about the relationship between $\vec{v}_e$ and $\vec{v}_i$. The implications of these assumptions will be discussed in the latter part of the paper. In the following section we proceed to discuss the physical implications of equation (26).

<u>The Electron Density Distribution with the Effect of a Variable Scale Height</u>

Equation (26) can be integrated along a field to provide the general solution

$$N(r, \theta) = f(r_o, \pi/2)\, e^{\frac{r \cot^2 \theta}{2H_i}} \tag{31}$$

However, if we treat T and m_i constant but recognize that g is proportional to $1/r^2$, H_i is proportional to r^2, and we obtain

$$N(r,\theta) = f(r_o, \pi/2)\, e^{\frac{-r \cos^2 \theta}{2H_i(r)}} \qquad e^{-\frac{r\cos^2\theta}{2H_i(r)}} \tag{32}$$

In both cases, $f(r_o, \pi/2)$ is an arbitrary function of height at the equator which cannot be determined by the equations of motion from which (31) or (32) are derived. The function $f(r_o, \pi/2)$ must therefore be given as a boundary condition in this problem and can only be determined emperically or by use of additional equations governing the physics of the problem.

Since (31) or (32) depend exclusively upon the equations of motion, it appears that an additional equation such as the continuity equation, should lead to the desired boundary condition. Unfortunately, as we ~~have discussed earlier~~ will show in the next section, the derivation and solution of the continuity equation depend upon a knowledge of $\vec{E}$. Thus, the complexity of the problem becomes quite formidable and it is difficult to anticipate a simple method of solution at this time.

Instead we depend upon an empirical type boundary condition, which may very well be the solution of the correct conti- nuity equation, to derive the explicit form of the electron density distribution.

The incorporation of a Chapman like boundary condition in (31) leads to the results obtained in GKS. Since such a boundary condition can only be considered as a rough approximation to the shape of the actual vertical electron density distribution at the equator, it is desirable to employ an analytic boundary condition which more closely resembles the true height profiles. Chandra (1962) has proposed a modified form of the Chapman function which includes the effect of variable scale height and which is found to fit the measured vertical distribution for electron density at mid-latitudes far more accurately than the simple Chapman form. We assume here that such a function also describes the electron density distribution at the equator. Thus we may write

$$f(r_o,\pi/2) = N_{r_{mo}} \exp \tfrac{1}{2} \left\{ 1 - \frac{r_o - r_{mo}}{H_o\left[1 - \alpha \exp\left(-\alpha \dfrac{(r_o - r_{mo})}{2H_o}\right)\right]} - \exp\left[- \frac{r_o - r_{mo}}{2H_o\left[1 - \alpha \exp\left(- \dfrac{\alpha(r_o - r_{mo})}{2H_o}\right)\right]} \right] \right\} \quad (33)$$

where H_o is the scale height of the ionic constituent and $N_{r_{mo}}$ is the value of electron density at the equatorial height r_{mo}. The parameter α, which is a measure of departure from simple Chapman function, is defined as

$$\alpha = \frac{H_o - H(r_{mo})}{H_o} \tag{34}$$

where $H(r_{mo})$ is that value of $H(r_o)$ at $r_o = r_{mo}$. Also, r_o is understood to be the radial height specifically at $\theta = \pi/2$.

If (32) is substituted into (31), we obtain

$$N(r, \theta) = N_{r_{mo}} \exp \tfrac{1}{2}\left\{1 - \frac{r \csc^2 \theta - r_{mo}}{H_o\left[1 - \alpha \exp\left(-\frac{\alpha(r \csc^2 \theta - r_{mo})}{2H_o}\right)\right]} - \frac{r \cos^2\theta}{H_o}\right.$$

$$\left. - \exp\left[-\frac{r \csc^2 \theta - r_{mo}}{2H_o\left[1 - \alpha \exp\left(-\frac{\alpha(r \csc^2 \theta - r_m)}{2H_o}\right)\right]}\right]\right\} \tag{35}$$

Equation (35) then provides a general expression for the electron density at all heights and co-latitudes provided that we are in a region where the effect of collision can be neglected.

The variation of $\left(\frac{N_r}{N_{r_{mo}}}\right)$ with co-latitude are shown in figures 1 - 3 for different values of α, h_{mo} and H_o. In the figures, we have converted radial height r to height measured from the earth's surface, h, where ~~For the purpose of numerical computation~~ the radius of the earth is taken to be 6370 km. Also, it is assumed that $H_o = H_i$. It is seen that the basic features of geomagnetic anomaly are unaltered by changing the various parameters. Further, in equation (35), if we identify the term $H_o\left[1 - \alpha \exp \frac{-\alpha}{2H_o}(\csc^2\theta - r_{mo})\right]$ with $\frac{1}{k}$

in GKS paper, it is seen that since α is positive, $kH_i>1$ for all heights. This explains why $KH>1$ gives better fit with the experimental data in the ~~latter~~ paper.

<u>Problems Involved in the Derivation of the Diffusion Equation</u>

In the previous sections we have seen how the equations of motion for electrons and ions are sufficient to obtain a theoretical description of the electron density distribution in the topside equatorial region of the ionosphere under equinox conditions. This has required us to make certain assumptions concerning collision frequencies or velocity components along field lines and also forced the application of an empirical boundary condition at the equator. In order to produce the empirical boundary condition theoretically and also obtain a solution which is valid in both the topside and bottomside equatorial F region, it is necessary to turn to the continuity equation for additional information. Using the explicit expressions for velocity which are derivable from the equations of motion, it is then possible to derive the diffusion equations associated with the ionosphere.

If we simply require total ambipolar diffusion to occur in the ionosphere so that $\vec{v}$ is independent of the electric field explicitly, and furthermore demand that in all regions concerned, the motion along field lines are much larger than the drifts normal to field lines, we must then invoke the additional constraint equation

$$\vec{v} \times \hat{B} = 0 \tag{36}$$

This leads to a hydrostatic distribution of electron density which doesn't agree with measured results, as has been demonstrated in C-I. A second approach (Kendall, 1962, and GS II) is the assumption that ambipolar diffusion exist only along field lines (see equation (19)). Thus, if we assume that the parallel velocity components of electron or ion velocity are equal and much greater than either of the unequal normal velocity components, we can write (20) as a good approximation for the entire velocity. In mathematical notation, we have

$$\vec{v} \cdot \hat{B} = |\vec{v}_{11}| = v_{11} \gg v_{e_\perp}, v_{i_\perp} . \qquad (37)$$

where $v_{e_\perp}$ and $v_{i_\perp}$ are the perpendicular components of electron and ion macroscopic ~~drift~~ velocity respectively. This implies that

$$\vec{v}_{11} \approx \vec{v} \qquad (38)$$

and provides us with a velocity expression independent of electric field. The general contention here has been that although (37) implies (38) so that we can write

$$Q - L = \nabla \cdot N \vec{v} \approx \nabla \cdot N \vec{v}_{11} \qquad (39)$$

as the steady state continuity equation, where Q and L are production and loss respectively, we no longer need invoke the equation of constraint since (36) is not exactly true. We can then substitute (20) into (39) and obtain the well known form of the two dimensional diffusion equation with no additional equation of constraint.

We wish to discuss this approach by first questioning the validity of (37), and then demonstrating that even if it were true, (28) cannot imply (39) without the additional inclusion of (36). This should demonstrate that the field line ambipolar diffusion approach with neglect of the normal velocity components is completely identical to the total ambipolar case in which the major component of drift velocities are assumed to lie along field lines. Thus, the results of the two approaches will be identical, leading to the conclusion that ambipolar diffusion in which the major component of drift velocity lies along a field line cannot be the correct physical model to describe the equatorial electron density distribution in the F region of the ionosphere.

Let us first consider (37). We have already seen that the assumption of diffusive equilibrium along a field line ($v_{11} = o$) leads to a correct description of the electron density in at least the topside region of the ionosphere. If this is the true model of the physical situation, then it is inconsistent with (37) and we cannot expect any results obtained using (37) to provide us with correct results concerning this region. If, on the other hand, the neglect of momentum transfer terms instead of diffusive equilibrium, can be attributed to small collision frequencies, (37) need not be violated. This might be a further justification for validating the collision frequency assumption instead of the diffusive equilibrium model. Unfortunately, as we approach the equator, we see from (20) that v_{11} approaches zero since both sin I and $\partial N/\partial\theta$ approach zero. The latter condition is based strictly on the empirical condition of symmetry about the equator. We therefore find that no matter how small $v_{e_\perp}$ and $v_{i_\perp}$ may be, there will always be a region about the equator in which (37) does not apply unless

$$v_{e_\perp} = v_{i_\perp} \equiv 0 \tag{40}$$

which is identical to the equation of constraint, (36).

We now return to the second question. That is, even if (37) were true, which could still be possible provided $v_{e_\parallel} \neq v_{i_\parallel}$, can we describe the electron density distributions in the entire region of the ionosphere by (39)? We note that

$$\nabla \cdot N\vec{v} = \vec{v} \cdot \nabla N + N \nabla \cdot \vec{v} = (\vec{v}_\parallel + \vec{v}_\perp) \cdot \nabla N + N \nabla \cdot (\vec{v}_\perp + \vec{v}_\parallel) \tag{41}$$

since

$$\vec{v} = \vec{v}_\parallel + \vec{v}_\perp \tag{42}$$

Now, in order to write (39), we must demand that

$$|\vec{v}_\parallel \cdot \nabla N + N \nabla \cdot \vec{v}_\parallel| >> |\vec{v}_\perp \cdot \nabla N + N \nabla \cdot \vec{v}_\perp| \tag{43}$$

We first note that the geomagnetic anomaly is symmetric about the equator during equinox. This implies that

$$\lim_{\theta \to \pi/2} \vec{v}_{\parallel} \cdot \nabla N = \lim_{\theta \to \pi/2} v_{\parallel} \hat{B} \cdot \nabla N = 0 \qquad (44)$$

so that in some region about the equator, (43) becomes

$$|N \nabla \cdot \vec{v}_{\parallel}| >> |N \nabla \cdot \vec{v}_{\perp} + \vec{v}_{\perp} \cdot \nabla N| \qquad (45)$$

Although (45) could be true for certain special cases, there is no a priori guarantee that (45) will be implied by (37) in general without the additional condition that $v_{\perp} = 0$. Thus, if we are to write (39) as a direct and general implication of (37), we are once again forced to employ the constraint equation $\vec{v} \times \hat{B} = 0$.

We cannot state that (40) holds in a very small region about the equator so that its effect outside this region can be neglected. The geomagnetic anomaly itself is a second order effect and we cannot expect to reproduce it by neglecting second order terms which are responsible for its existence.

We therefore find that if ambipolar diffusion exists in the ionosphere, and if it is restricted to the field line direction, we cannot assume (37) without imposing an additional constraint equation. Furthermore, (37) does not generally imply (39) in the ionosphere with or without ambipolar diffusion unless the constraint equation is also employed. However, since (37), (39), and the assumption of ambipolar diffusion along a field line do not provide to the correct description of equatorial electron density, N, we must conclude that these assumptions are not valid in a theory leading to a description of the electron density distribution in the ionosphere.

Kendall (1962), and Rishbeth, Lyon and Peart (1963), have attempted to numerically integrate (30) derived from (20) and (37), without invoking (26). They have been unable to obtain the correct description of the geomagnetic anomaly and have therefore concluded that diffusion may not be a very important physical process governing the empirical distribution of electron density. However, on the basis of the discussion presented in this section, it now appears that the physical assumptions used in deriving the form of (39) used in their work may not be valid, which simply implies that the diffusion equation is far more complicated than originally believed. Since (19) and (37) are no longer valid, we can no longer equate electron and ion velocities to eliminate electric field. Instead, we must write separate continuity equations for electrons and ions and describe the behavior of electric field before it is possible to obtain the correct theoretical description of the geomagnetic anomaly from the diffusion equation.

It may appear that the results presented in GS II are also not valid for the reasons discussed above. However, a closer inspection of GS II shows that no new information was obtained from the solution of continuity equation than that already available from the equations of motion. The equation discussed in GS II was simply

$$\nabla \cdot N \vec{v}_{\parallel} = 0$$

$$\nabla N\vec{v}_{11} = 0 \qquad (46)$$

Where the explicit production and loss terms were neglected in obtaining the series solution. Furthermore, as shown in GKS, the equation of motion leading to (26), whether derived assuming $\vec{v}_{11}$ = o or making assumptions concerning collision terms, has the identical solution to that obtained from (46) in GS II. (For $\vec{v}_{11}$ = 0, (39) 46 obviously cannot give any new information.) This explains why the empirical boundry condition was necessary to obtain a non-arbitrary solution using (39) 46 in GS II.

NO NEW ¶ We should point out however, that ~~in~~ solutions of (46) 39 making use of explicit production and loss terms would not give correct results ~~at least~~ in the equatorial region for the reasons discussed in this section.

The Distribution of the Neutral Atmosphere

In this section we will show that when the drag forces are not negligible, as would be the case in the lower F-region and E-region, it is possible to study the behavior of the neutral atmosphere without imposing any restriction on the velocities of the various constituents. To obtain the necessary starting equation, we first sum (3), (4), and (5):

$$-\nabla(p_e + p_i + p_n) + (n_n m_n + N_i m_i) \vec{g} + \vec{J} \times \vec{B} = 0 \qquad (47)$$

where we have once again used (14). The component of (47) along the direction of magnetic field is then

$$[-\nabla(p_e + p_i + p_n) + (m_n n_n + m_i N)\vec{g}] \cdot \hat{B} = 0 \qquad (48)$$

Comparison of (47) and (48) shows that the net force due to pressure gradient and gravity of all particles is perpendicular to the magnetic field and balanced by a current flow force. Next, using (7), (8), (9), (18), and (28), we have

$$-\frac{\partial^{\rightarrow}(n_n + 2N)}{\partial r} + \frac{\tan\theta}{r}\frac{\partial^{\rightarrow}(n_n + 2N)}{\partial\theta} + \frac{n_n}{H_n} + \frac{N}{H_i} = 0 \qquad (49)$$

where H_n is the scale height of the neutral atmosphere.

Since $N << n_n$ and $H_i \approx H_n$, we can write

$$\frac{n}{H_n} + \frac{N}{H_i} \approx \frac{n}{H_n} + \frac{N}{H_n} = \frac{n + 2N}{H_n} \qquad (50)$$

where the suffix on n has been dropped for simplicity. Then, (49) becomes, in total derivative form,

$$\frac{d(n + 2N)}{dr} + \frac{n + 2N}{H_n} = 0 \qquad (51)$$

Integration of (51) can be obtained along the field line giving

$$n(r, \theta) \approx n + 2N \approx g(r_o, \pi/2)\, e^{-\int_{r_o}^{r}\frac{dr}{H_n}} \qquad (52)$$

where $g(r_o, \pi/2)$ is an arbitrary function of height at the equator and r_o is defined in (27). If we now demand that the radial distribution of the neutrals obey the normal hydrostatic law at the equator, so that

$$g(r_o, \pi/2) = n_{oo}\; e^{-\int_{r_{oo}}^{r_o}\frac{dr}{H_n}} \qquad (53)$$

where n_{oo} is the neutral number density at height r_{oo} on the equator, then

$$n(r) = n_{oo}\, e^{-\int_{r_{oo}}^{r} \frac{dr}{H_n}}, \qquad (54)$$

a result which is entirely independent of θ. If, on the other hand, $g(r_o, \pi/2)$ is perturbed in any manner from the exact hydrostatic equilibrium case, we will obtain a distribution for n which does depend on θ. The origin of this angular dependence on the neutrals may seem somewhat surprising until we realize that in selecting a functional form for $g(r_o, \pi/2)$, any deviation in the equatorial neutral distribution from hydrostatic equilibrium must arise due to collisions between neutrals and geomagnetically controlled charged particles. Thus, if the collisions between neutrals and charged particles are sufficiently large to make the momentum transfer forces between charged and neutral particles important, the neutrals will begin to tend toward the angular distribution of the geomagnetically controlled particles. This can also be seen from (3), where it is obvious that we will not obtain the exact hydrostatic distribution in a region when the terms on the left hand side become important. On this basis, we might expect to observe angular variations of the neutral distribution in the bottomside regions of the ionosphere where charged-neutral particle interactions become important.

Conclusions

From the discussion and results of this paper, we have shown the following:

1. From the equations of motion, it is possible to derive an expression for electron-density distribution along a field line either by assuming diffusive equilibrium along the direction of the magnetic field or by neglecting the drag forces arising due to collision. The latter assumption appears to be more realistic in the topside of the ionosphere. In either case, it is necessary to assume a distribution at a certain point in the radial direction in order to obtain a complete description of the electron density distribution.

2. We have provided a more accurate formula for the representation of the equinox geomagnetic anomaly than that produced in GKS. Since the empirical boundary condition equation used herein has been shown by Chandra (1962) to fit nearly all vertical profiles of electron density measured to date, we can safely assume that the proper selection of parameters in this formula will lead to a reasonable reproduction of the anomaly in any region of the ionosphere where either simultaneous diffusive equilibrium of ions and electrons occurs, or where interactions of neutrals with charged particles are small because of infrequent collisions.

3. The derivation of the diffusion equation commonly employed to obtain a theoretical description of the electron density distribution in the F region ionosphere depends upon the assumptions $\vec{v}_{e_{\parallel}} = \vec{v}_{i_{\parallel}} >> \vec{v}_{e_{\perp}}$ or $\vec{v}_{i_{\perp}}$. Attempts to numerically integrate the diffusion equation derived in this manner have not described the geomagnetic anomaly with the correct behavior. We have now been able to show that the assumptions made in the deviation are inconsistent with experimental data unless the

additional equation of constraint $\vec{v} \times \hat{B} = 0$ is also employed.

Since the numerical integration results are based on an incomplete derivation of the diffusion equation, it is understandable why such results are in poor agreement with measurement. Furthermore, we find that the inclusion of the constraint equation leads to a horizontally stratified electron density distribution which also is incapable of describing the geomagnetic anomaly. We must therefore conclude that the diffusion equation derived in the above manner is based on assumptions which do not fit the true physical description of the ionosphere and which lead to the neglect of the very physical parameters responsible for the geomagnetic anomaly.

4. A study of the neutral atmosphere distribution has led us to the conclusion that geomagnetic control of neutrals occurs in any region of the ionosphere where interactions of neutrals with charged particles become important. Since this is most likely to occur in the lower F region of the ionosphere, we suggest that such geomagnetic control of the neutrals might be observable in this region.

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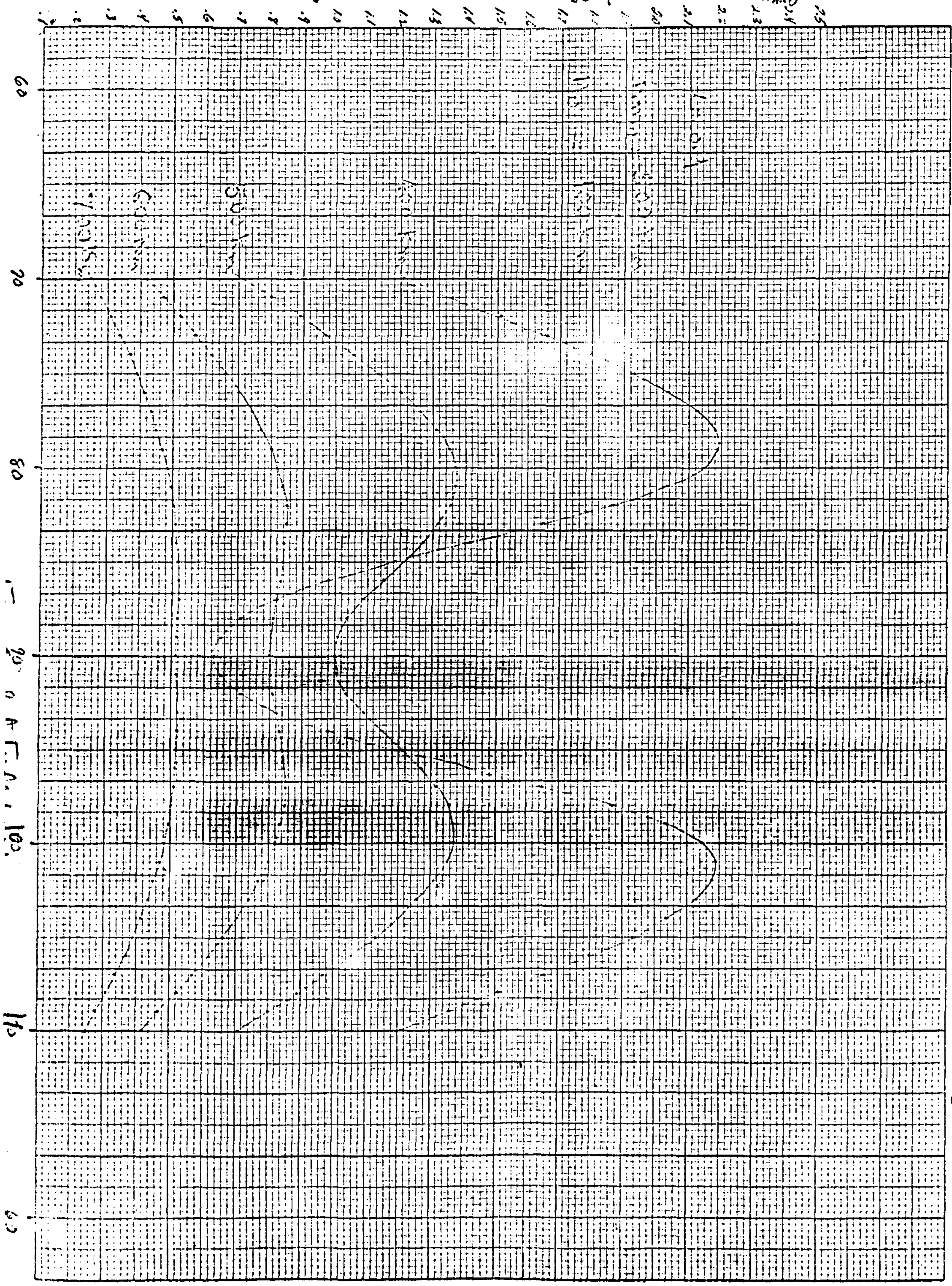
Relative Electron Density N(h,θ)/N_hmo



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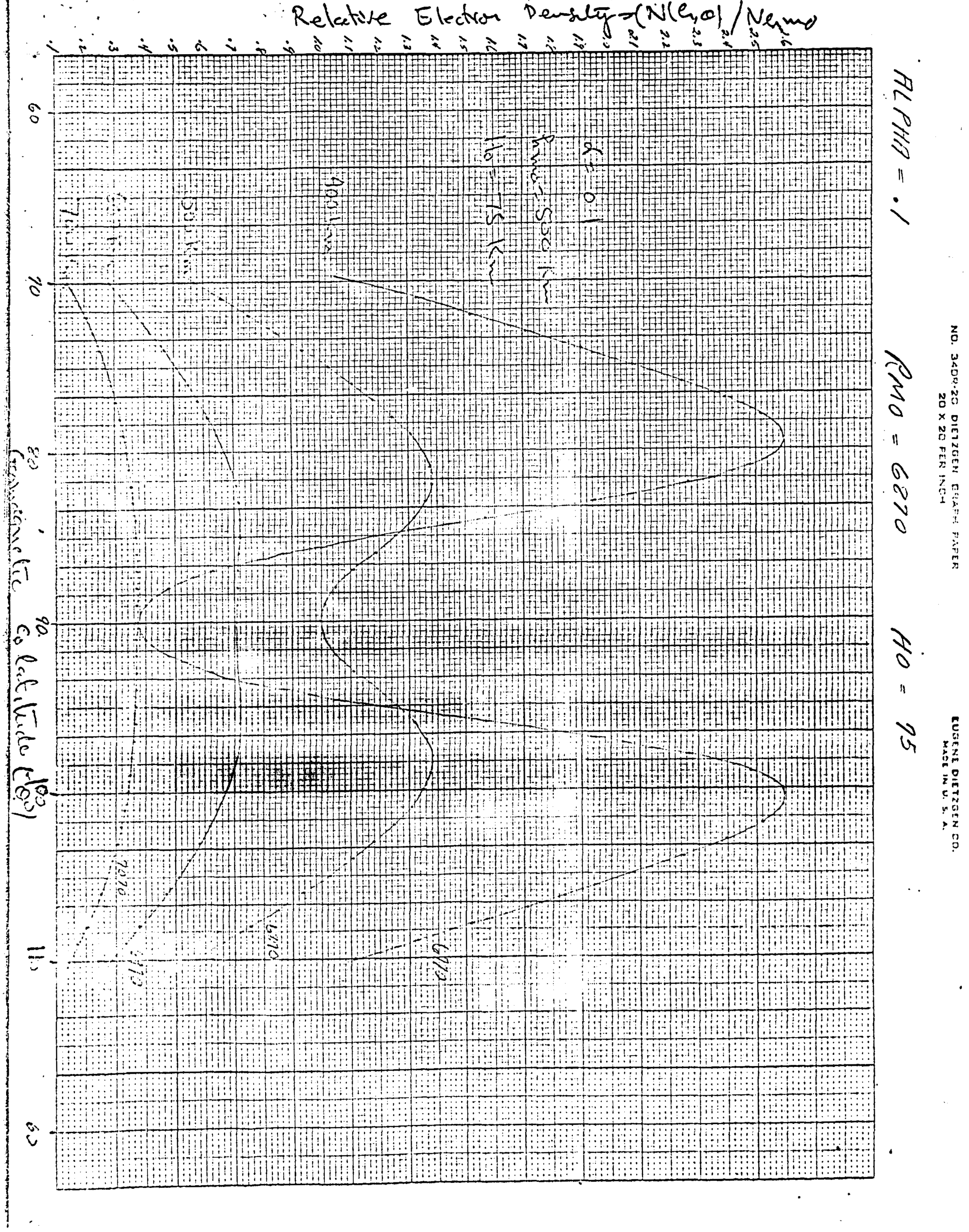

Relative Electron Density
ALPHA = .1
RMO = 6870
HO = 95
α = 0.1
ho = 75 km
400 km
500 km
60
70
80
90
100
110
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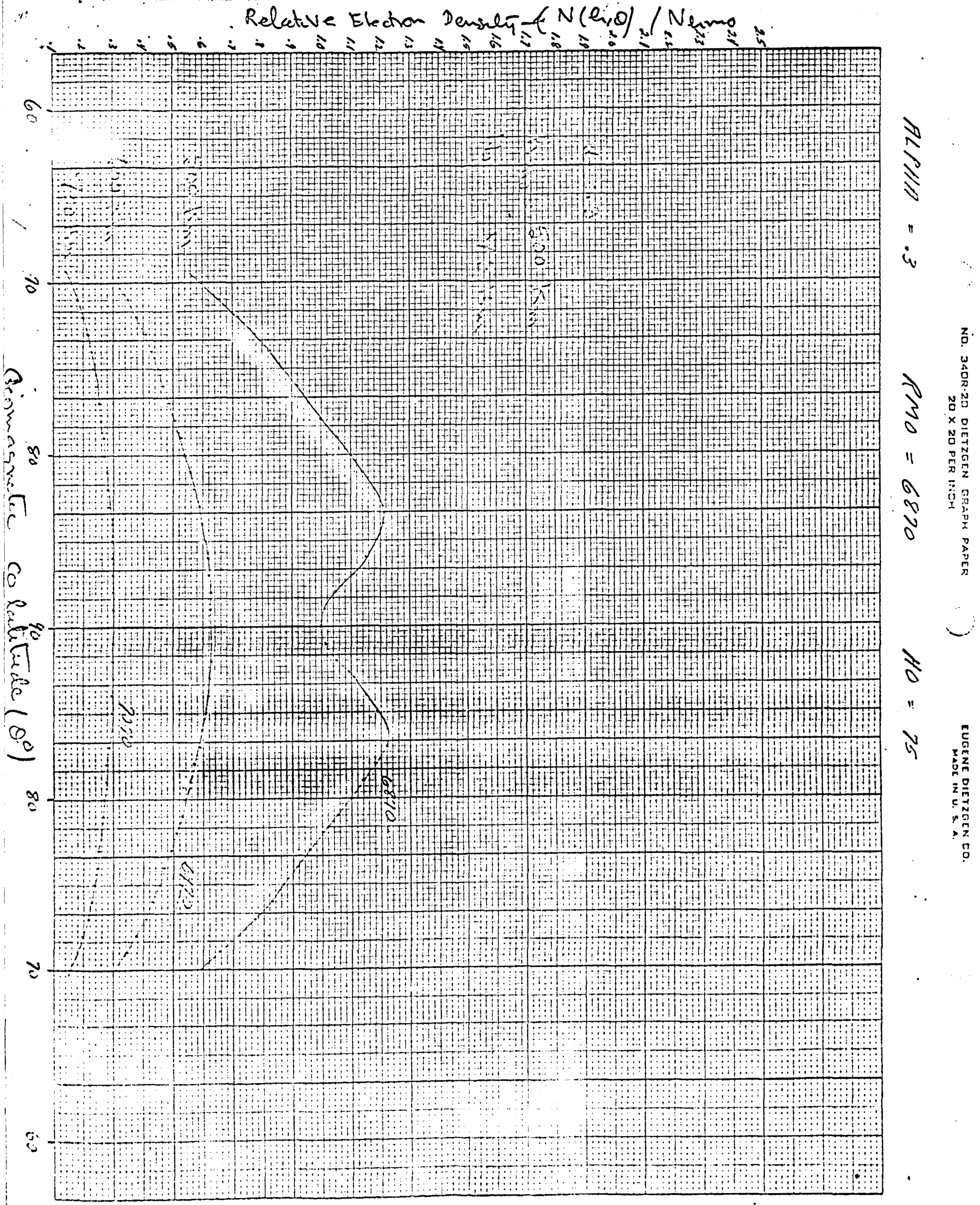

Relative Electron Density N(h,θ) / Nhmo
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2 2.3 2.4 2.5
60
70
80
90
80
70
60
Geomagnetic Co-latitude (θ°)
ALPHA = .3
RMO = 6870
HO = 75
6870
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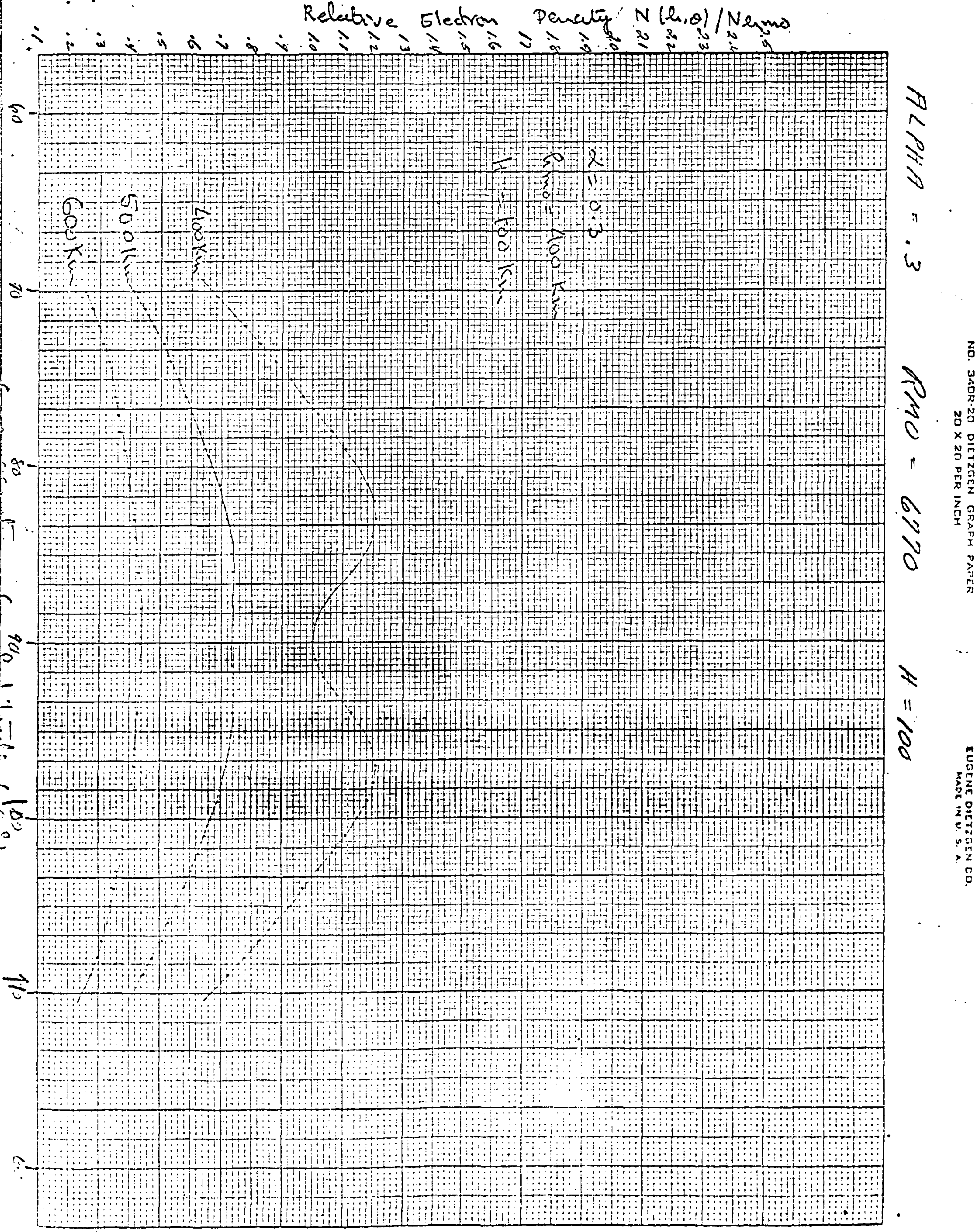

Relative Electron Density N(h,0)/Nemo
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2 2.3 2.4 2.5
60
70
80
90
100
110
120
600 Km
500 Km
400 Km
H = 100 Km
Rmo = 400 Km
α = 0.3
ALPHA = .3
RHO = 6770
H = 100
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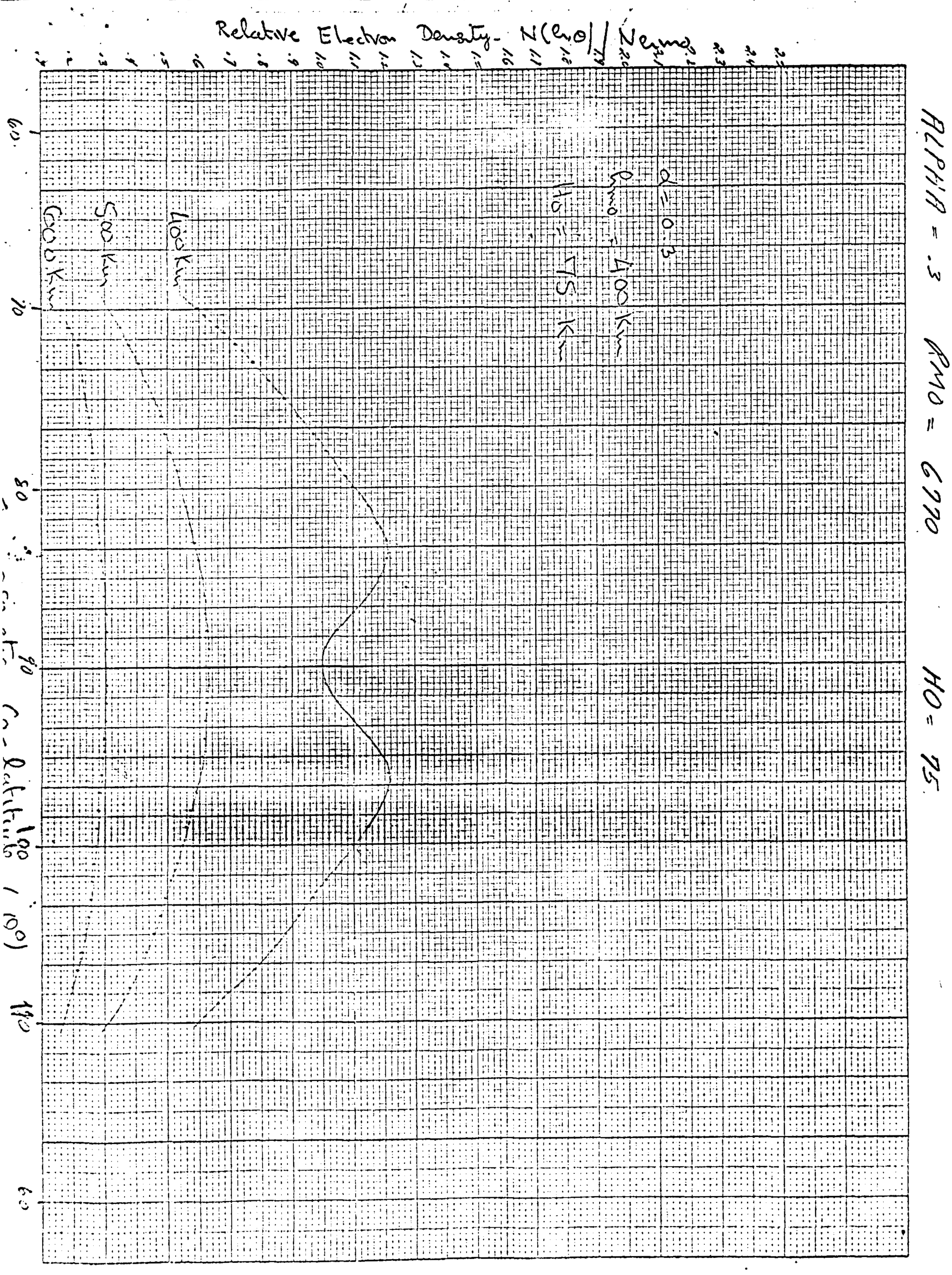

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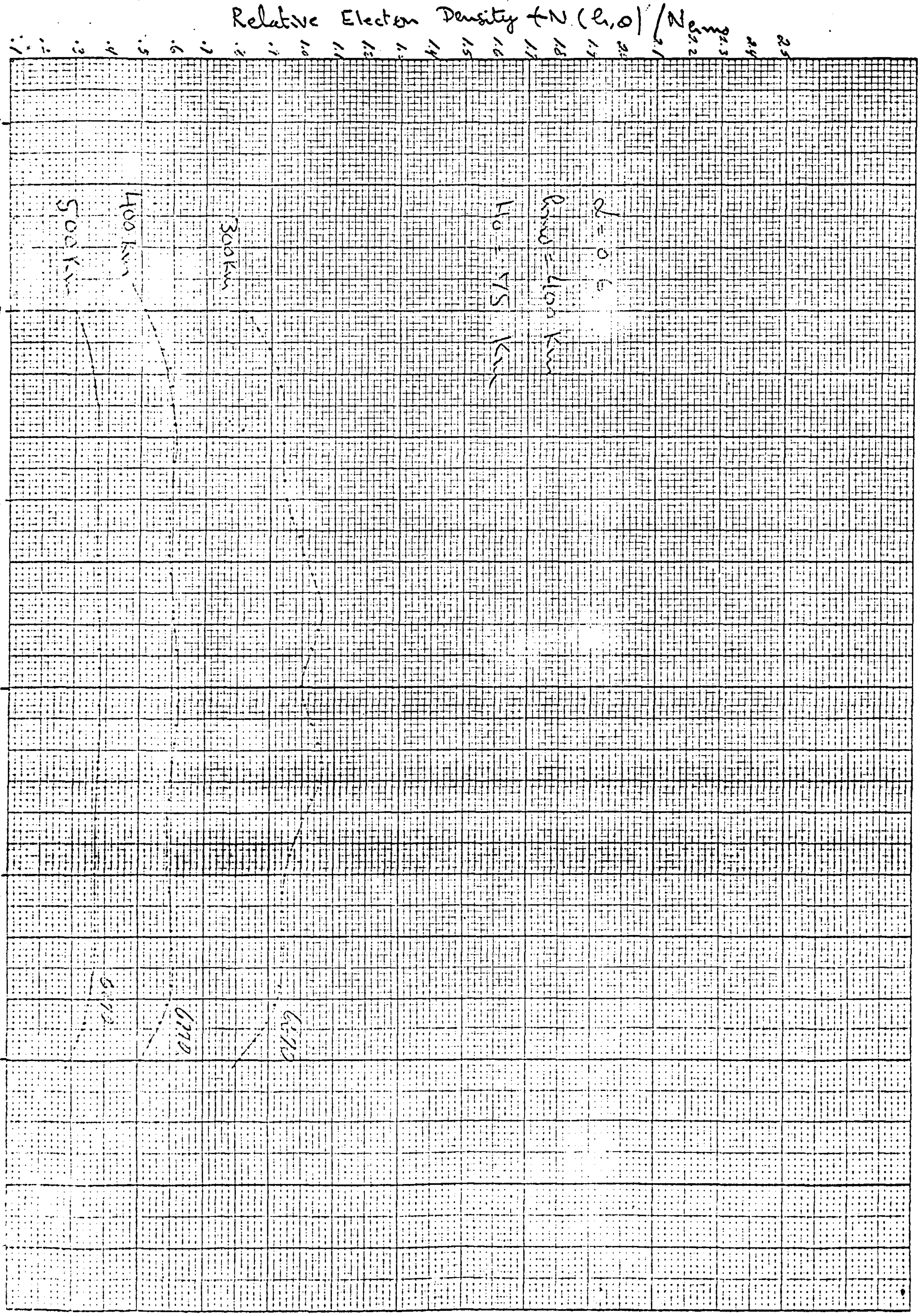

ALPHA = .6 RMO = 6770 HO = 75

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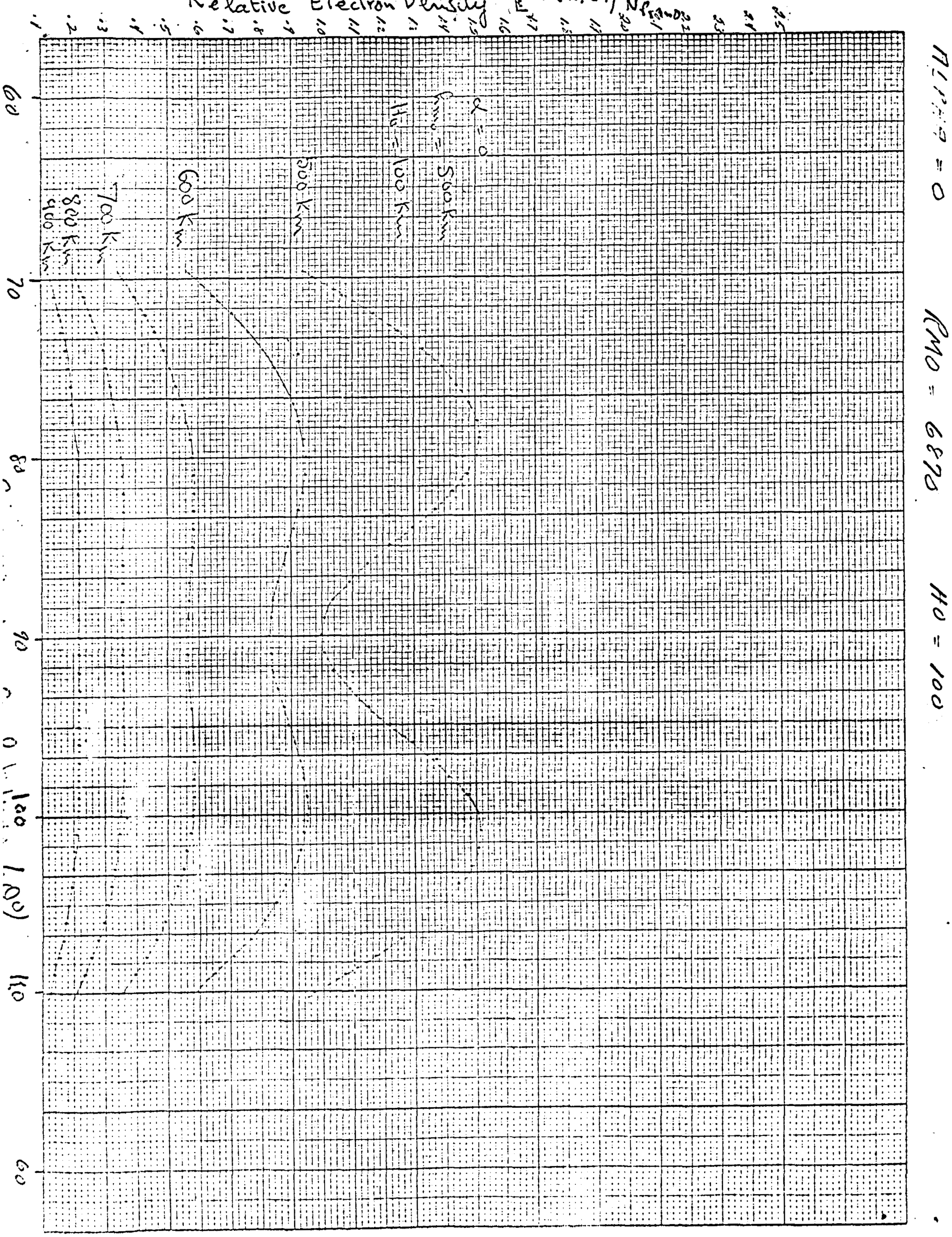
Relative Electron Density EN(h,θ)/N_STANDARD



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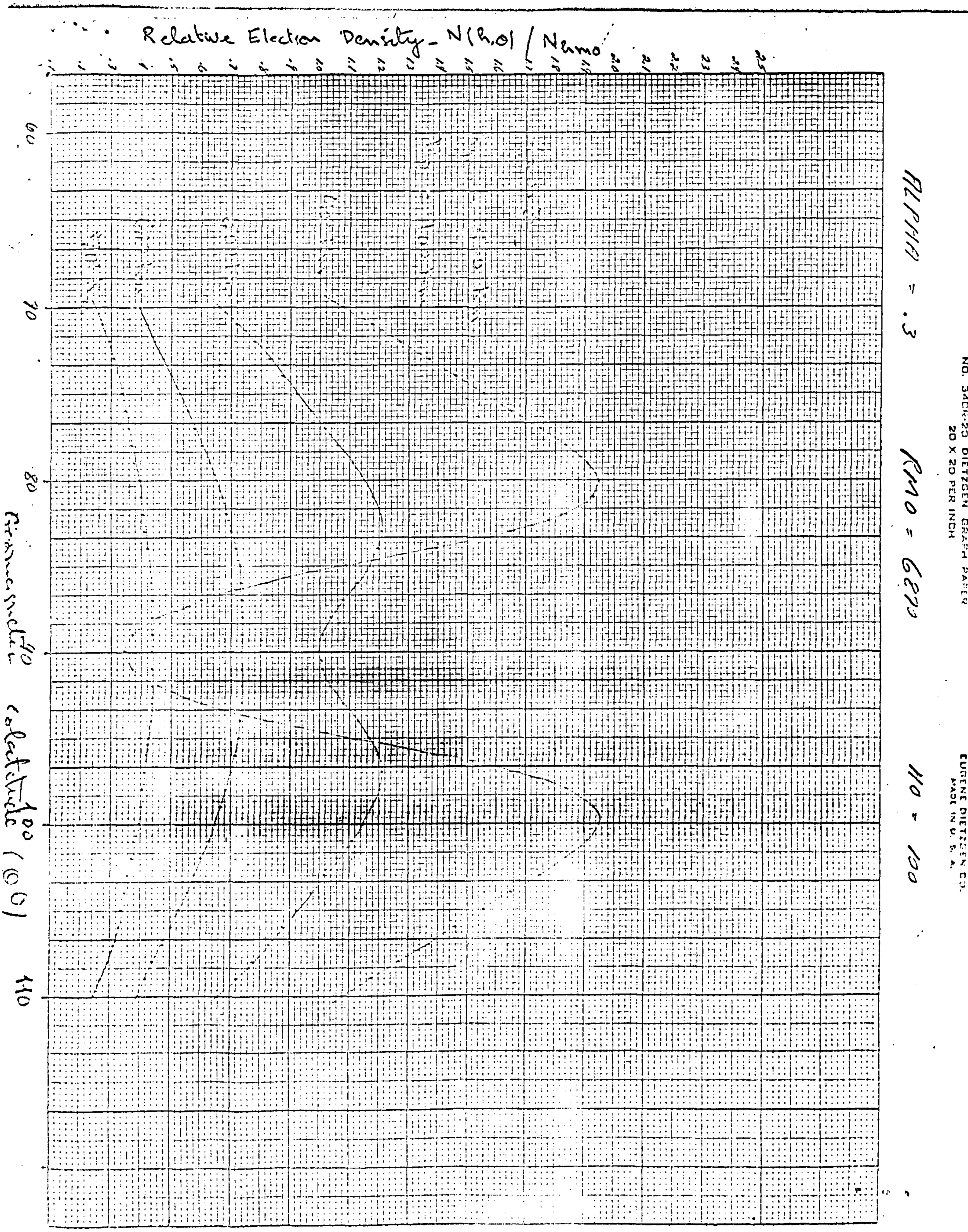
Relative Electron Density - N(h,0) / Nemo
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2 2.3 2.4 2.5
60
70
80
90
100
110
Geomagnetic colatitude (θ°)
ALPHA = .3
RMO = 6270
H0 = 100
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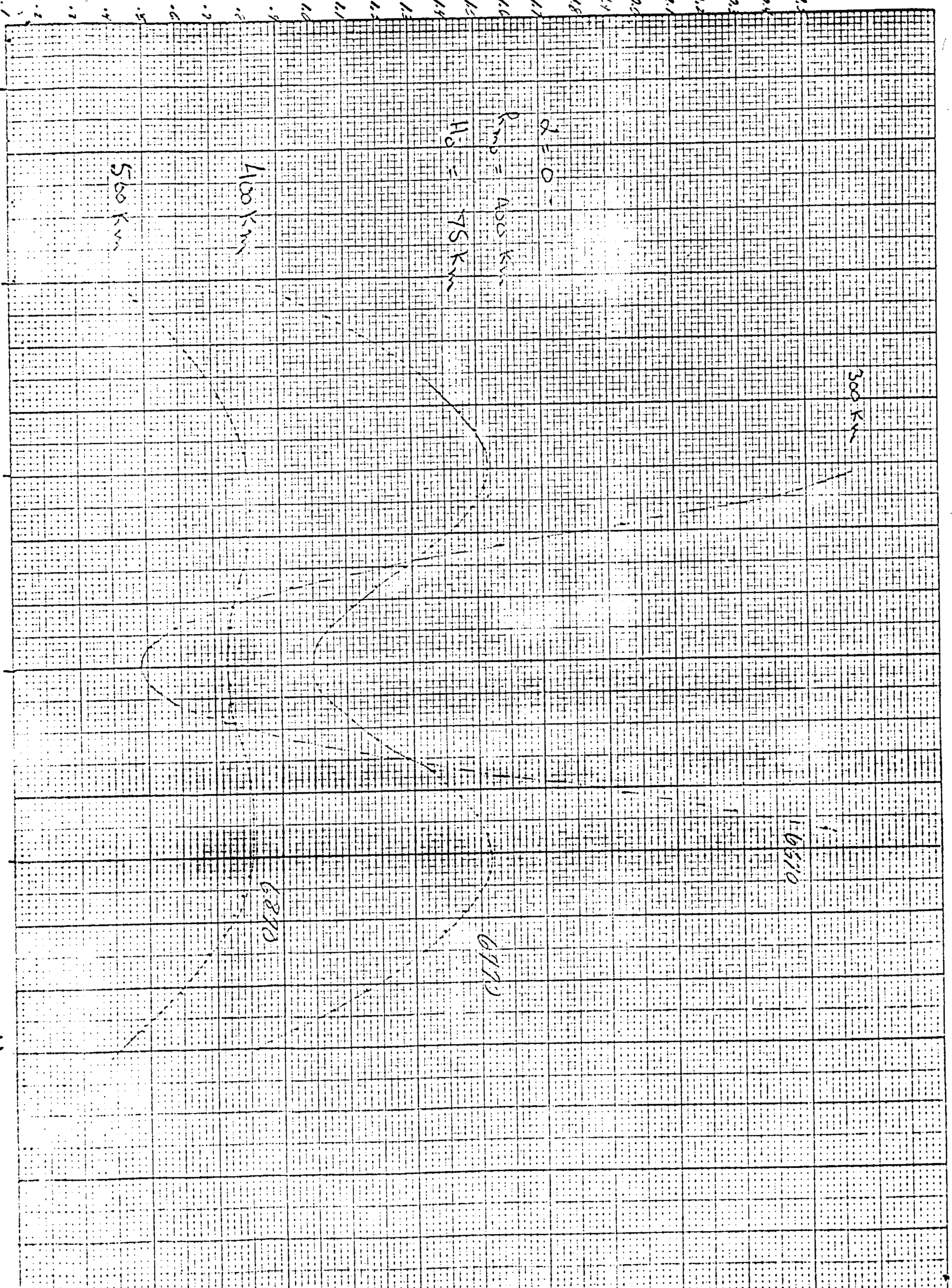
Relative Electron Density N(h,0)/Nhmo



ALPHA = 0 RMO = 6770 HO = 75

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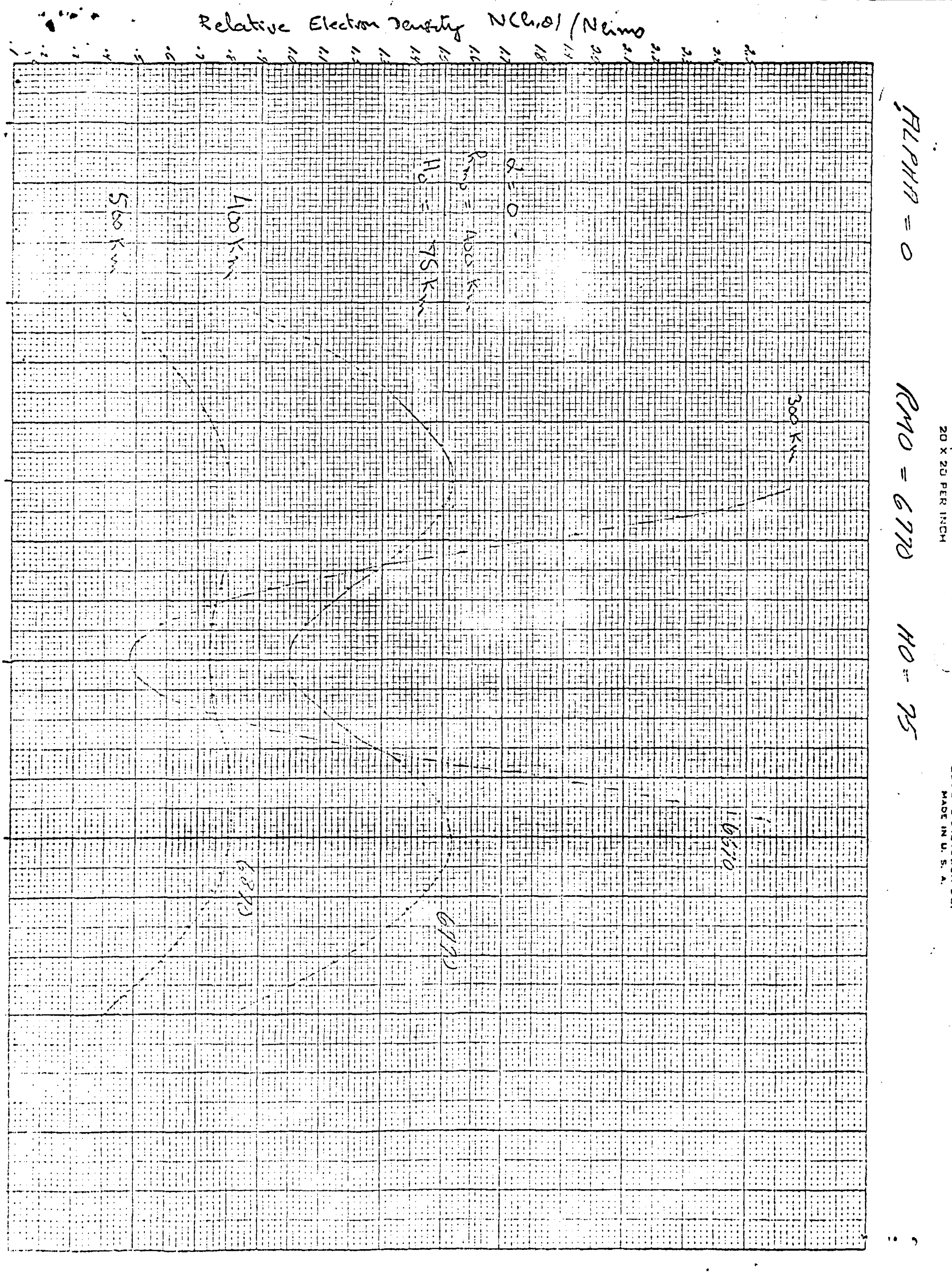

Relative Electron Density N(h,0)/N(hmo)
ALPHA = 0
RMO = 6770
HO = 75
500 Km
400 Km
α = 0
Rmo = 400 Km
Ho = 75 Km
300 Km
6510
6770
6770
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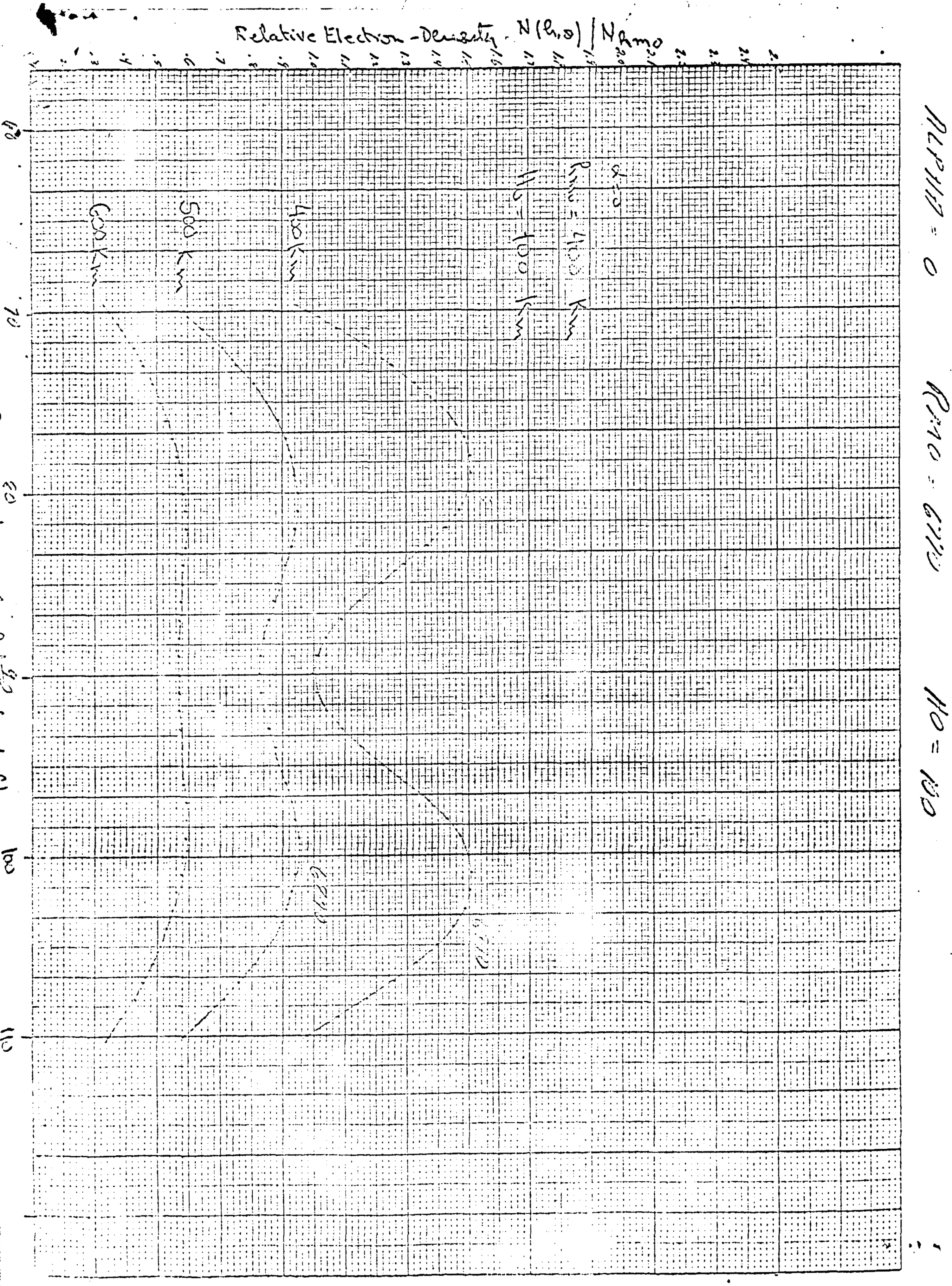

Relative Electron-Density $N(h,0)/N_{hmo}$
ALPHA = 0
RMO = 670
HO = 100
α = 0
Rmo = 400 Km
Ho = 100 Km
400 Km
500 Km
600 Km
670
670
60
70
80
90
100
110